

DP entry test: problems for practice

Write without brackets or negative indices:

a $(mn)^{-2}$

b mn^{-2}

c $\left(\frac{x}{5y^2}\right)^{-3}$

Write the answers to the following in scientific notation:

a The speed of light in a vacuum is about $2.998 \times 10^8 \text{ m s}^{-1}$.

Assuming 1 year ≈ 365.25 days, determine how far light travels in:

i 1 hour

ii 1 day

iii 1 year.

b How long does it take for light to travel:

i 1 m

ii 1 cm

iii 1 mm?

c In air, light travels at $2.989 \times 10^8 \text{ m s}^{-1}$ and sound travels at 343.2 m s^{-1} .

How many times faster is light than sound?

For each of the following sets P and Q , decide whether $P \subseteq Q$:

a $P = \{5, 6, 7, 8\}$, $Q = \{1, 2, 3, 4, 5, 6, 7\}$

b $P = \{\text{multiples of 4 between 10 and 30}\}$, $Q = \{\text{even numbers between 0 and 40}\}$

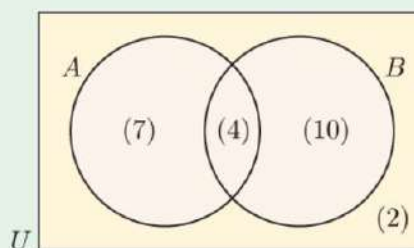
How many elements are there in:

a A

b B

c $A \cup B$

d neither A nor B ?



400 families were surveyed. It was found that 90% had a TV set, and 60% had a computer. Every family had at least one of these items. How many families had both a TV set and a computer?

Expand and simplify:

a $x(x^2 - 3) + 5(x - 4)$

b $(a + b)(a - b) - (a + 2b)(a - 2b)$

Fully factorise:

a $x^2 - 5x - 66$

b $2x^2 + 20x - 78$

c $4x^2 - 8x - 21$

Simplify:

a $2\sqrt{3} + 6\sqrt{5} - 3\sqrt{3} - 4\sqrt{5}$

b $\frac{\sqrt{6}}{3} - \frac{\sqrt{6}}{4} + \frac{2\sqrt{6}}{5}$

Express with integer denominator:

a $\frac{8}{\sqrt{2}}$

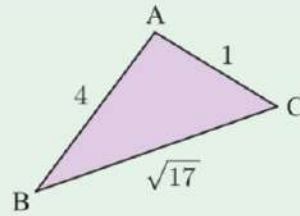
b $\frac{15}{\sqrt{3}}$

c $\frac{\sqrt{3}}{4 + \sqrt{2}}$

d $\frac{5}{6 - 2\sqrt{3}}$

A drinks stall sells small, medium and large cups of fruit drink for €1.50, €2 and €2.50 respectively. In one morning, three times as many medium cups were sold as small cups, and the number of large cups sold was 140 less than the number of medium cups. If the total of the drink sales was €1360, how many of each size cup were sold?

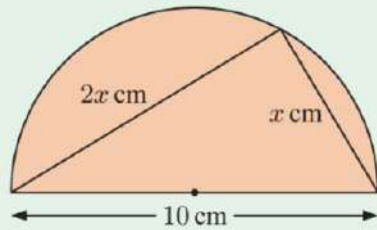
Is this triangle right angled?
Explain your answer.



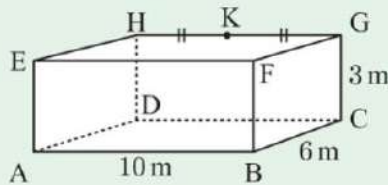
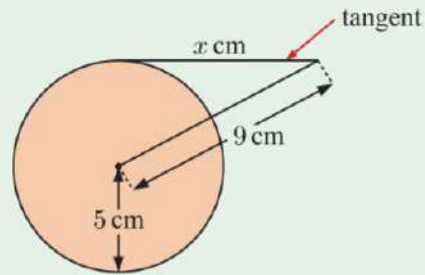
A circle has a chord of length 10 cm. The shortest distance from the circle's centre to the chord is 5 cm. Find the radius of the circle.

Find x :

a



b



A room is 10 m by 6 m by 3 m.
Find the shortest distance from:

- a** E to K
- b** A to K.

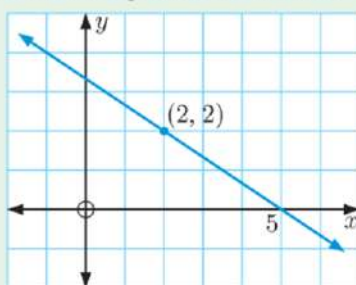
Find the midpoint of the line segment joining $A(-2, 3)$ to $B(-4, 3)$.

Find the distance from $C(-3, -2)$ to $D(0, 5)$.

Find the equation of the line:

- a** with gradient -2 and y -intercept 7
- b** passing through $(-1, 3)$ and $(2, 1)$
- c** parallel to a line with gradient $\frac{3}{2}$, and passing through $(5, 0)$.

Find the equation of the line:



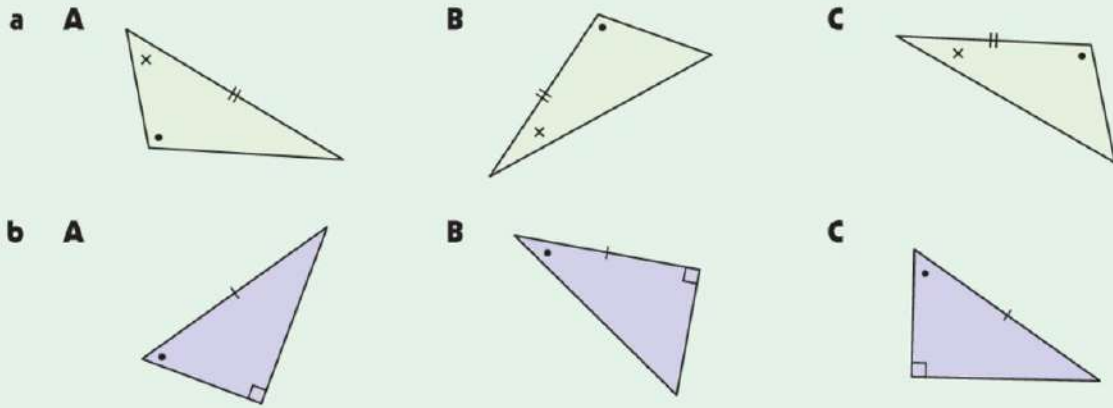
Flour is sold in 5 kg and 2 kg packets. The 5 kg packets cost €2.75 each and the 2 kg packets cost €1.25 each. If I bought 67 kg of flour and the total cost was €38.50, how many of each size of packet did I buy?

Find the coordinates of any point where these line pairs meet:

a $y = 3x + 2$
 $y = 3x - 1$

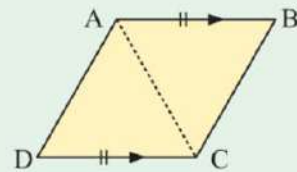
b $2x - 3y = 18$
 $4x + 5y = -8$

In each set of three triangles, two are congruent. State which pair is congruent giving a reason for your answer. The triangles are not drawn to scale, but contain correct information.

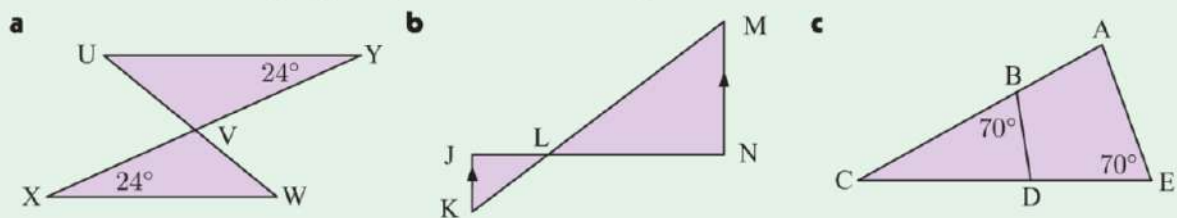


Consider the quadrilateral ABCD.

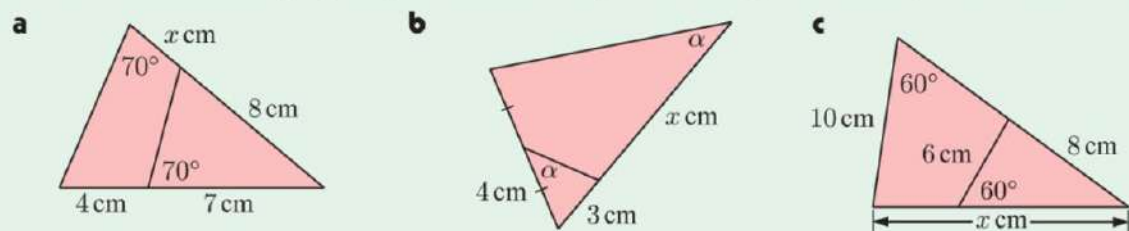
- Show that triangles ABC and CDA are congruent.
- Hence deduce that ABCD is a parallelogram.



Show that the following figures possess similar triangles.

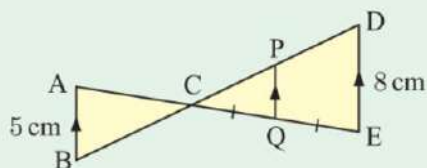


In each of the following figures, establish that a pair of triangles is similar, and hence find x :



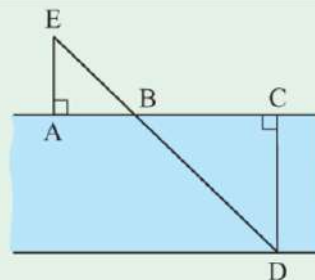
$\triangle ABC$ has an area of 15 cm^2 .

- Find the area of $\triangle CDE$.
- Find the area of PQED.



A, B, and C are pegs on the bank of a canal which has parallel straight sides. C and D are directly opposite each other. $AB = 30 \text{ m}$ and $BC = 140 \text{ m}$.

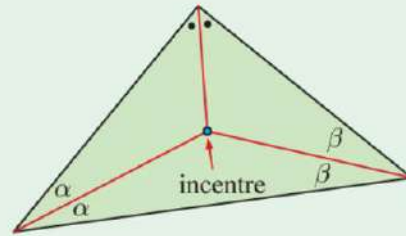
When I walk from A directly away from the bank, I reach a point E, 25 m from A, such that E, B, and D line up. How wide is the canal?



The three angle bisectors of a triangle meet at a point called the **incentre** of the triangle.

Show that the incentre is equidistant from each edge of the triangle.

Hint: Draw a perpendicular line from each edge to the incentre.



A sphere of lead with radius 10 cm is melted into 125 identical smaller spheres. Find the radius of each new sphere.

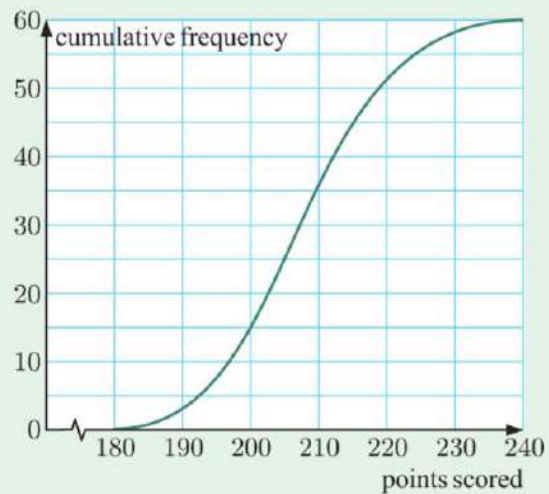
A class of 20 students was asked “How many bedrooms are there in your house?” The following data was collected:

3 2 3 2 2 4 3 4 2 3 2 1 2 2 3 2 4 2 3 2

- a** Is the data discrete or continuous? **b** Are there any outliers in the data?
c Construct a dot plot to display the data.

Consider the set of data: 17 14 9 12 23 14 12 18 9 15 6 14 21 13 10

- a** Find the:
i mode **ii** mean **iii** median **iv** range
v upper and lower quartiles **vi** interquartile range.
b Draw a box-and-whisker plot to display the data.



The cumulative frequency graph illustrates the points scored by competitors in a ski aerials competition.

- a** How many competitors took part in the competition?
b What percentage of competitors scored less than 200 points?
c Estimate the median score.

If $p = 5$, $q = -3$, and $r = 6$, evaluate:

- a** $\frac{r}{q}$ **b** $\frac{p - q}{p + q}$ **c** $\frac{\sqrt{p^2 - 16}}{r - q}$ **d** $\frac{p + 2q - 2r}{r^2 - p^2}$

Simplify:

- a** $\frac{2a - 2b}{b - a}$ **b** $\frac{5x - 15}{3x - x^2}$ **c** $\frac{16 - x^2}{2x - 8}$

Simplify:

a $\frac{7x-14}{x} \times \frac{3}{x-2}$

b $\frac{t^2-3t}{6t+6} \times \frac{t+1}{4t-12}$

Simplify:

a $\frac{9}{n} \div 6$

b $\frac{7}{3x-6} \div \frac{x+5}{x^2-2x}$

Solve for x :

a $x^2 - 4x - 21 = 0$

b $4x^2 - 25 = 0$

c $6x^2 - x - 2 = 0$

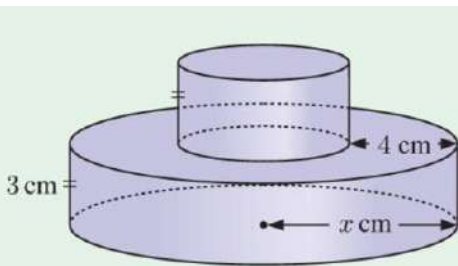
Solve by completing the square: $x^2 + 6x + 4 = 0$

Use the quadratic formula to solve for x :

a $2x^2 - 3x - 2 = 0$

b $3x^2 + 4x - 5 = 0$

c $\frac{x+4}{x-2} = \frac{5x}{x-1}$

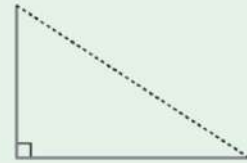


The volume of the solid alongside is $174\pi \text{ cm}^3$.
Find x .

A straight length of wire is 20 cm long. It is bent at right angles to form the two shorter sides of a right angled triangle.

a If the triangle's area is 30 cm^2 , find:

- the length of each side
- the triangle's perimeter.

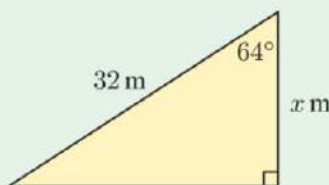


b Is it possible for a right angled triangle with shorter sides made from a 20 cm length of wire to have an area of 51 cm^2 ? Explain your answer.

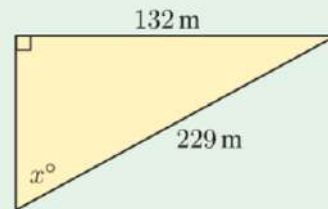
Solve for x : $\frac{1}{x-3} + \frac{3}{x+1} = 1$

Find the value of x :

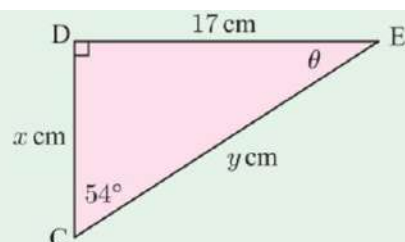
a

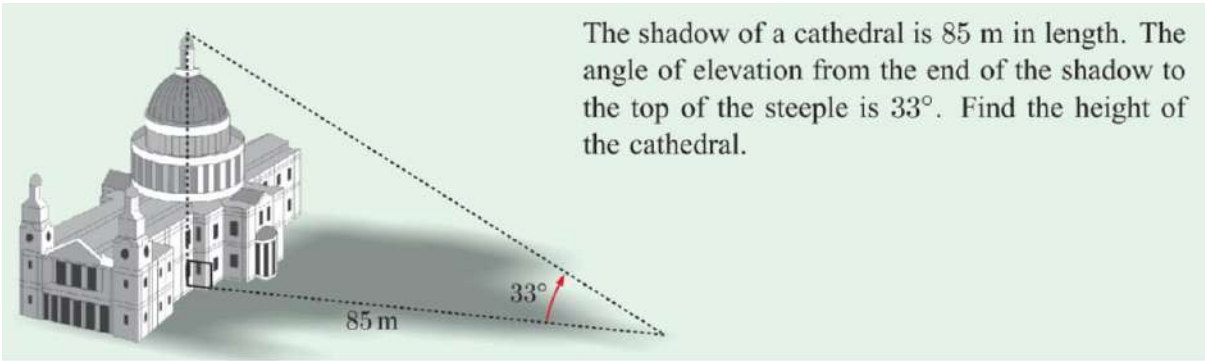


b



Find the measure of all unknown sides and angles in triangle CDE.



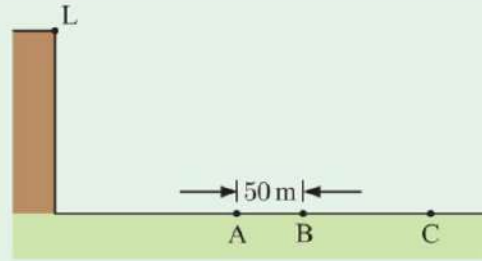


A taxi travels 8 km south, then 3 km west. Find the bearing of the taxi's finishing point from its starting point.

Laura is at the top of a scenic lookout. Her friends Ariel and Briannah are at ground level, and are 50 m apart.

The angle of depression from L to A is 36° , and the angle of depression from L to B is 29° .

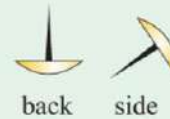
- Find the angle of elevation from B to L.
- Find the height of the lookout.
- The girls' car is at C, at an angle of depression of 20° from L. How far is the car from Ariel?



A coin and a pentagonal spinner with sides labelled A, B, C, D, and E are tossed and spun simultaneously. Illustrate the possible outcomes using a 2-dimensional grid.

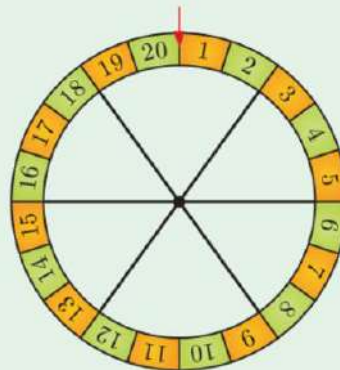
When a box of drawing pins was dropped onto the floor, 49 pins landed on their backs, and 32 landed on their sides. Estimate, to 2 decimal places, the probability of a drawing pin landing:

- on its back
- on its side.



A wheel numbered 1 to 20 is spun. Find the probability that the result is:

- 13
- a multiple of 3
- greater than 11.



On a particular day, 500 people visited a carnival. 300 people rode the Ferris wheel, and 350 people rode the roller coaster. Each person rode at least one of these attractions.

- Display this information on a Venn diagram.
- Find the probability that a randomly chosen person:
 - rode the Ferris wheel, but not the roller coaster
 - rode the roller coaster, given that they rode the Ferris wheel.

A container holds 3 banana iceblocks, 4 chocolate iceblocks, and 2 raspberry iceblocks. George randomly selects an iceblock from the container and eats it. His brother Oliver then randomly selects an iceblock from the container.

- a** Draw a tree diagram to display the possible outcomes.
- b** Find the probability that the brothers selected the same type of iceblock.

The two-way table alongside describes the books on Elizabeth's bookshelf.

	Biography	Novel	Reference
Softcover	3	22	6
Hardcover	5	7	4

- a** How many books are on Elizabeth's bookshelf?
- b** Find the probability that a randomly selected book is:
 - i** a softcover book
 - ii** a hardcover reference book

Make x the subject of:

a $mx + n = 3p$

b $\frac{7}{y} = \frac{5}{x}$

Make x the subject of the formula $y = \frac{2x - 3}{x - 2}$.

For $f(x) = 3x - x^2$, find:

a $f(2)$

b $f(-1)$

c $f(x - 3)$

Consider the function $g(x) = x^2 + 2x$. Find:

a $g(2)$

b $g(3x)$

c x such that $g(x) = 15$.

Write down the first five terms of the sequence:

- a** starting at 8, and increasing by 5 each time
- b** starting at 19, and decreasing by 7 each time
- c** whose n th term is the n th cubic number.

Find the first four terms of the sequence with recursive formula $u_1 = 2$, $u_n = 2u_{n-1} + 4$, $n \geq 2$.

Find the unknowns given that the following sequences are arithmetic:

a 9, 17, \square , 33, 41,

b 27, \square , 15, \triangle , 3,

Find k given that $k - 2$, $2k - 1$, and $4k - 6$ are consecutive terms of an arithmetic sequence.

Write down the common ratio for these geometric sequences:

a 250, 200, 160, 128,

b 4, -12, 36, -108,

Find the sum of:

a $21 + 25 + 29 + 33 + \dots$ to 20 terms

b $40 + 34 + 28 + 22 + \dots$ to 30 terms.

Examine the matchstick pattern:

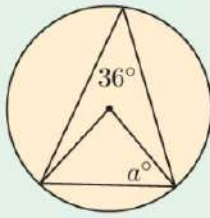


How many matchsticks make up the n th diagram?

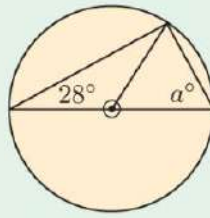
Find the coordinates of the points of intersection of the graphs with equations $y = \frac{3}{x}$ and $y = 2x - 1$.

Find the value of a :

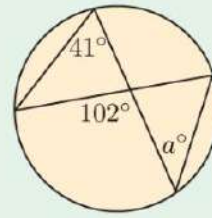
a



b



c



A, B, and C are three points on a circle. The bisector of angle BAC cuts [BC] at P, and the circle at Q. Prove that $\widehat{APC} = \widehat{ABQ}$.

Consider the quadratic function $y = -2(x - 1)(x + 3)$.

a Find the:

i direction the parabola opens

ii y -intercept

iii x -intercepts

iv equation of the axis of symmetry.

b Sketch the function, showing all of the above features.

a Write the quadratic $y = x^2 - 4x + 10$ in the form $y = (x - h)^2 + k$.

b Hence sketch the function, stating the coordinates of the vertex.

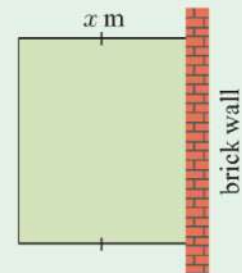
A vegetable gardener has 40 m of fencing to enclose a rectangular garden plot where one side is an existing brick wall. Suppose the plot is x m wide as shown.

a Show that the area enclosed is given by

$$A = -2x^2 + 40x \text{ m}^2.$$

b Find x such that the vegetable garden has the maximum possible area.

c Find the maximum possible area.



Solve for x and graph the solution:

a $5x - 11 > -7$

b $9 \leq 4 - 2x$

Solve for x :

a $(x + 2)(x + 6) < 0$

b $x^2 + 5x \geq 36$

c $2x^2 - 15 < 7x$

Following an outbreak of the *Ebola* virus, a rare and deadly haemorrhagic fever, medical authorities begin taking records of the number of cases of the fever. Their records are shown below.

Days after outbreak (n)	2	3	4	5	6	7	8	9	10	11
Diagnosed cases (d)	8	14	33	47	80	97	118	123	139	153

a Produce a scatter plot of d against n .

b Plot the point (\bar{n}, \bar{d}) on the scatter plot, and draw the line of best fit by eye.

c **i** Use the graph to predict the number of diagnosed cases on day 14.

ii Is this predicted value reliable? Give reasons for your answer.

Convert:

- a** 5.3 km to m **b** 20 000 cm² to m² **c** 5 m³ to cm³ **d** 0.48 L to cm³

Convert:

- a** 3200 mm to m **b** 15 ha to m² **c** 3600 cm³ to mm³ **d** 4.5 kL to m³

Find the area of:

- a** a sector of radius 10 cm and angle 120°
b a right angled triangle with base 5 cm and hypotenuse 13 cm
c a rhombus with sides of length 5 cm and one diagonal of length 4 cm.

- a** Determine the length of fencing around a circular playing field of radius 80 metres if the fence is 10 metres from the edge of the field.
b If the fencing cost £12.25 per metre of fence, what was the total cost?